**Title**: Intro to infinite series through Zeno’s paradoxes

**Author**: Jenna Jerez

**Topic**: Zeno’s paradoxes, sums of infinite series

**Overview**: In this activity, students will be exposed to some of Zeno’s paradoxes. Through discussion, students will be led to construct the concept of an infinite series and consider whether such series converge.

**Objective**: Students will construct a concept of infinite series and discover that $\sum\_{i=1}^{\infty }\frac{1}{2^{i}}$ converges to 1.

**Materials**: “Zeno’s Paradox” geogebra worksheet

**Activity**:

Explain Zeno’s paradox of “Achilles and the Tortoise” to students and allow them time to examine the applet.

Achilles can run with great speed. In fact, he is the fastest among the Greek warriors. Achilles is to run a foot race against a very slow opponent—a tortoise. In light of the sluggish character of the tortoise, Achilles allows the tortoise a head start half-way to the finish line…Zeno argues that Achilles will not win the race, or rather that Achilles *cannot* win the race. In fact, Zeno maintains that Achilles will not even be able to surpass the tortoise no matter how fast he runs…by the time Achilles reaches the starting point of the tortoise, the tortoise will have advanced a certain distance; Achilles will again have to traverse the distance between himself and the tortoise. Yet by the time Achilles reaches that point, the tortoise will have advanced yet again, and Achilles will have to traverse that distance. And so on. (Chow 5-6)

Discuss as a class the students’ thoughts about the paradox. Do they agree? Do they disagree? Why?

Now introduce Zeno’s paradox “The Dichotomy” (also mentioned in the applet).

Achilles is once again on the race course. Zeno maintains that Achilles will not be able to reach the finish point of the race course… In order for Achilles to reach the finish point, he must first traverse half the distance to it. Again, before he reaches the finish point, Achilles must traverse half the distance between the half-way point and the finish point. Achilles must then traverse half the distance between that point and the finish point. In other words, before Achilles can traverse the entire race course, he must first run half the race course, then a remaining quarter, then a remaining eighth, etc…Notice that the number of runs that Zeno contends Achilles must perform has the form of an infinite progression: $\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{1}{16}…$ An infinite series has no last member. Thus, as Zeno argues, Achilles’ run has no last member and he can never complete his last task, and so can never reach the goal of the race course. (Chow 6-7)

Again, lead the class in a discussion about the paradox. (Consider allowing a student to act out the situation described in the second part of the applet if time allows or if students are confused.) Can Achilles ever reach his destination?

Have a student write on the board the sum of the distances that Achilles is running (i.e. $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+…$). Ask the students if they think it adds up to a specific number. Allow time for responses. Direct students to begin adding the numbers. They should notice that they are getting closer and closer to 1.

Give a brief description of a limit. Explain to students that $S=\lim\_{n\to \infty }\frac{1}{2^{n}}=1$.

Reference: Chow, S. J. (2006). *Zeno's paradoxes and problems with infinity in the physical world.*(Order No. MR19456, Dalhousie University (Canada)). *ProQuest Dissertations and Theses,*, 117-117 p. Retrieved from http://search.proquest.com/docview/304952668?accountid=14761. (304952668).